Toward New Practical Digital Signature Scheme based on Lattices

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Joint work with
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Hash-and-Sign

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Basic Framework

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  - Hard to find SK from VK and pairs of (m, s)
  - Hard to find "small" s satisfying $As = H(m)$

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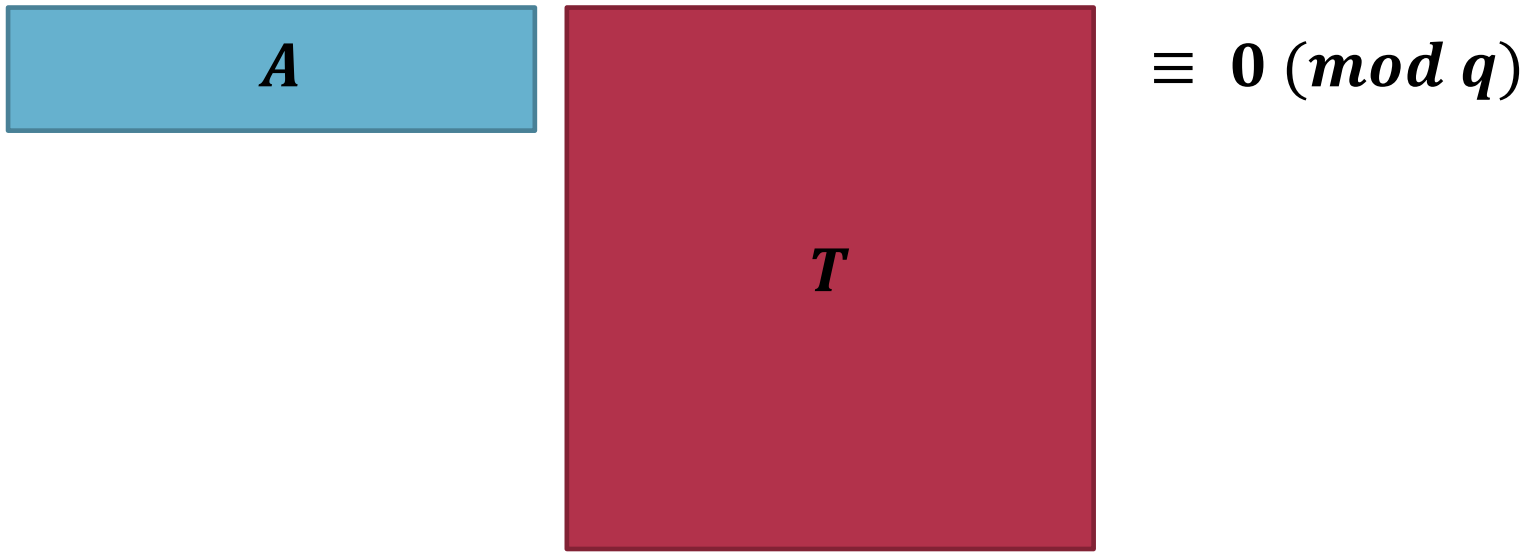

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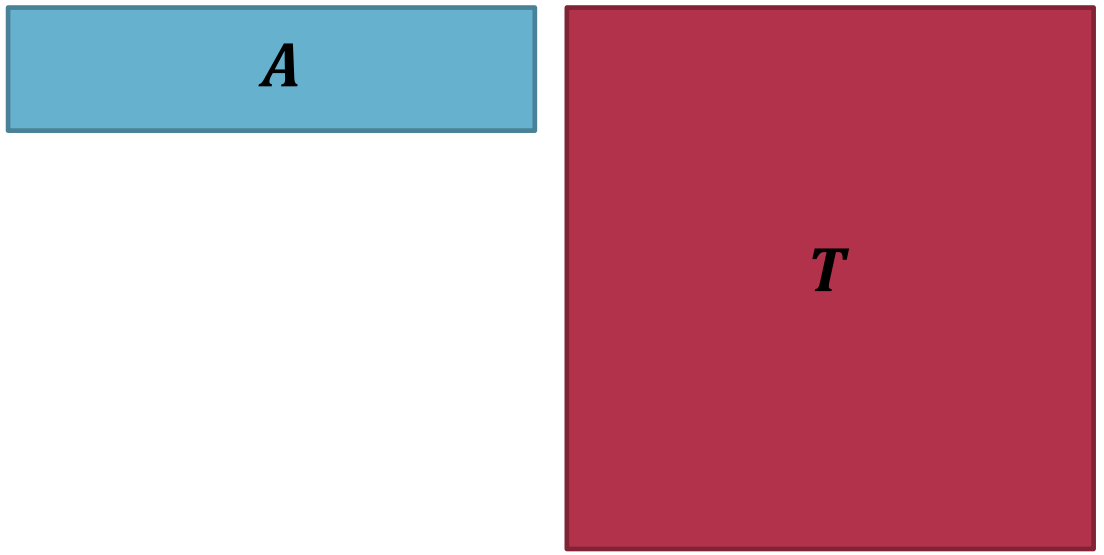

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satisfying

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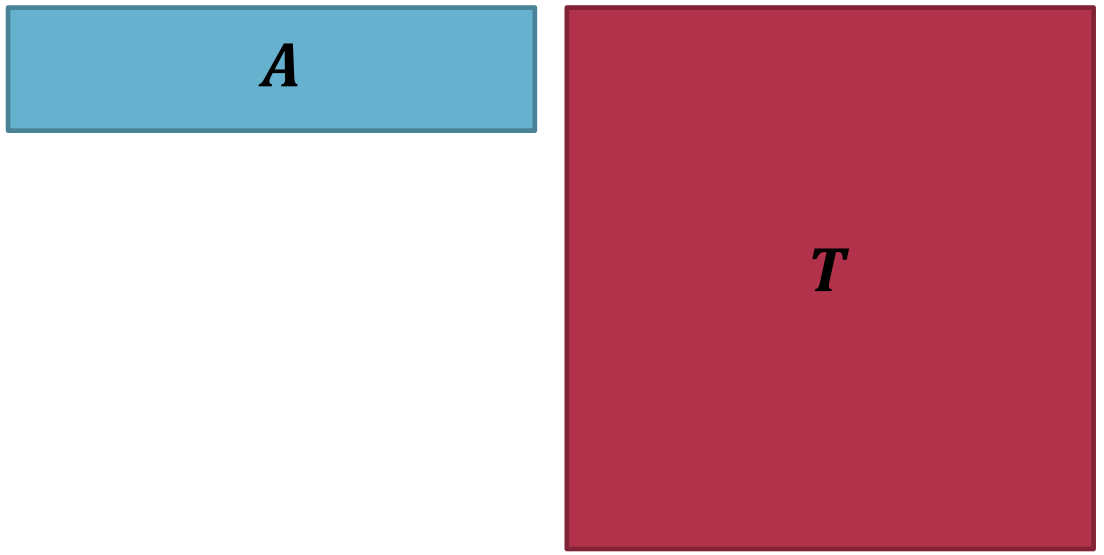

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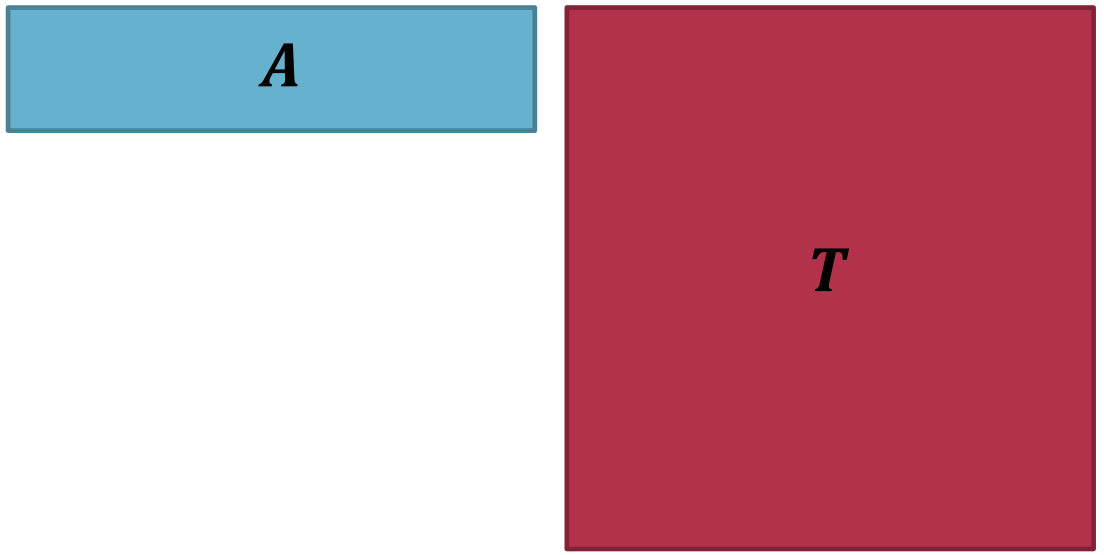

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  - Drawback in Practicality: too large parameters (due to statistical property and w/o ring structure)

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  - For "small" $f, g \in R_q = Z_q[X]/(X^n + 1)$ and $h = \frac{g}{f}$, one can generate "small" $F, G \in R_q$ s.t.

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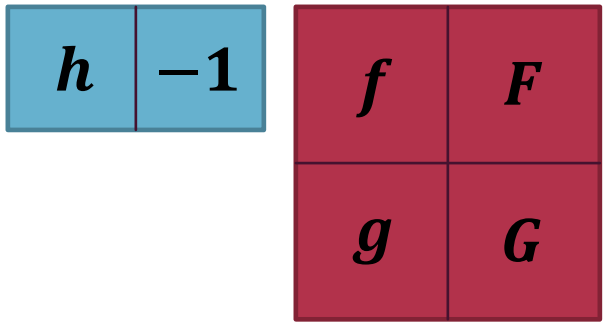

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satisfying

$$fG - gF = q$$

High-level structure of Trapdoor Lattice

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Our Goal:

To publish a (ring) lattice $A \in R_q^{k \times d}$ w/ short basis T of $\Lambda_q^{\perp}(A)$, which does NOT give any information of T computationally

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"How can we interpret the NTRU trapdoor lattice in high-level?"

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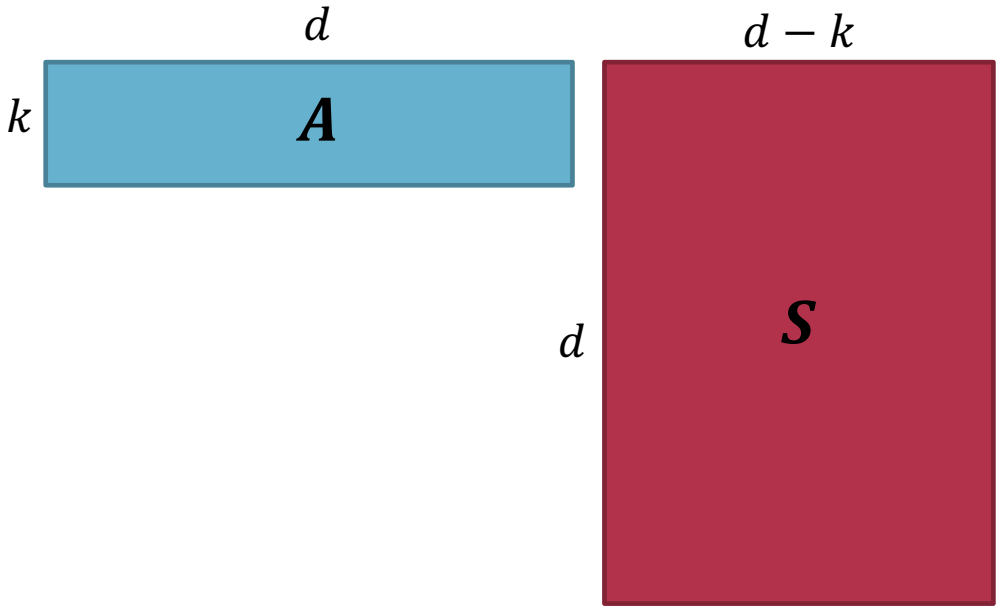

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- $d - k$ = Rank(T) over R_q < d

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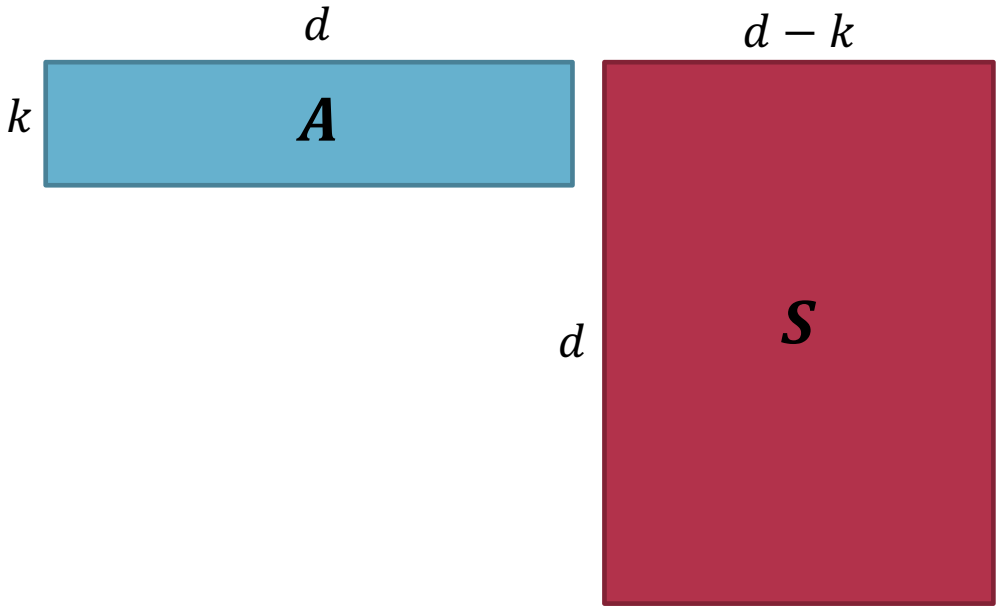

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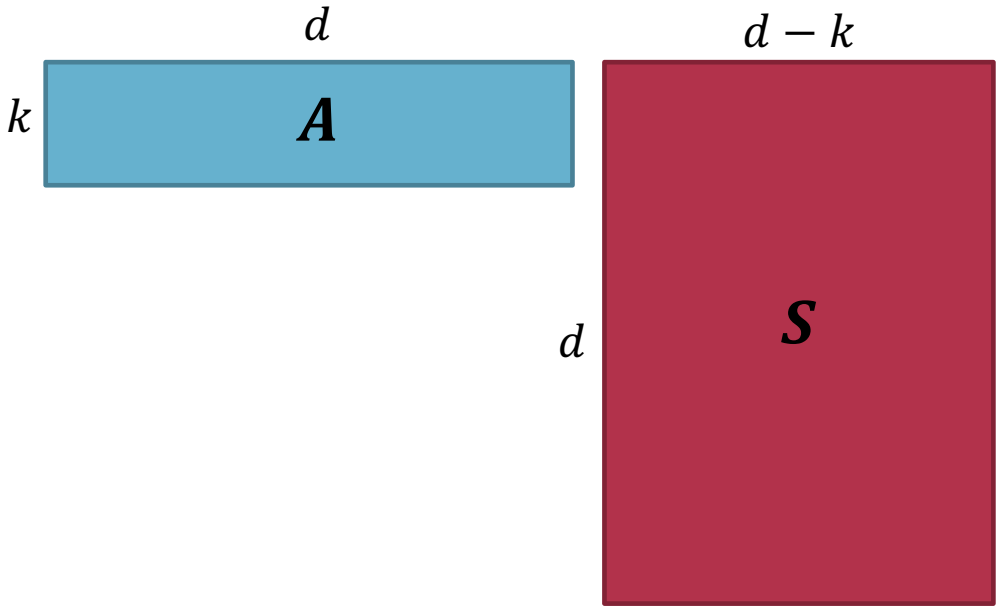

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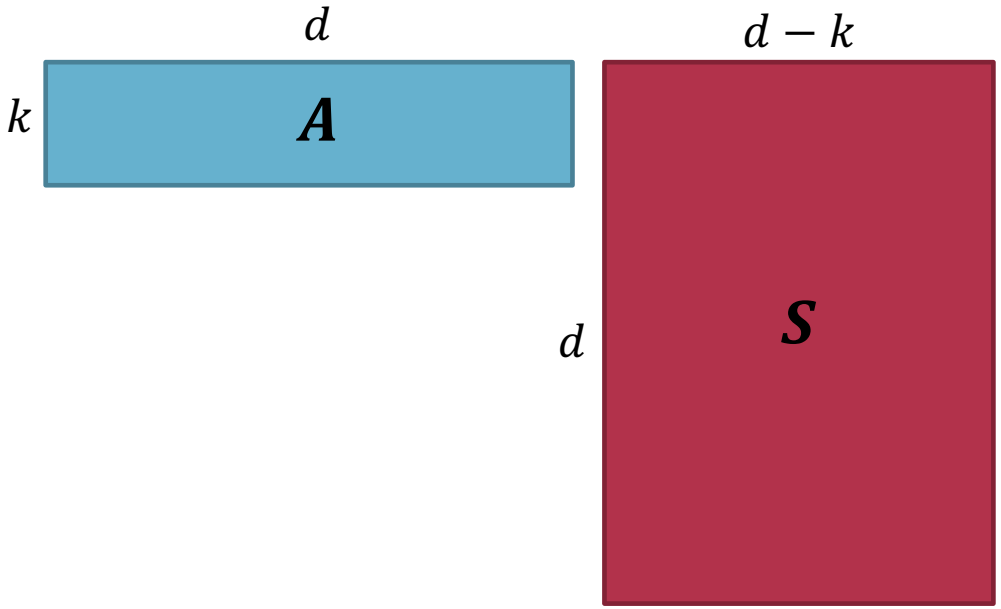

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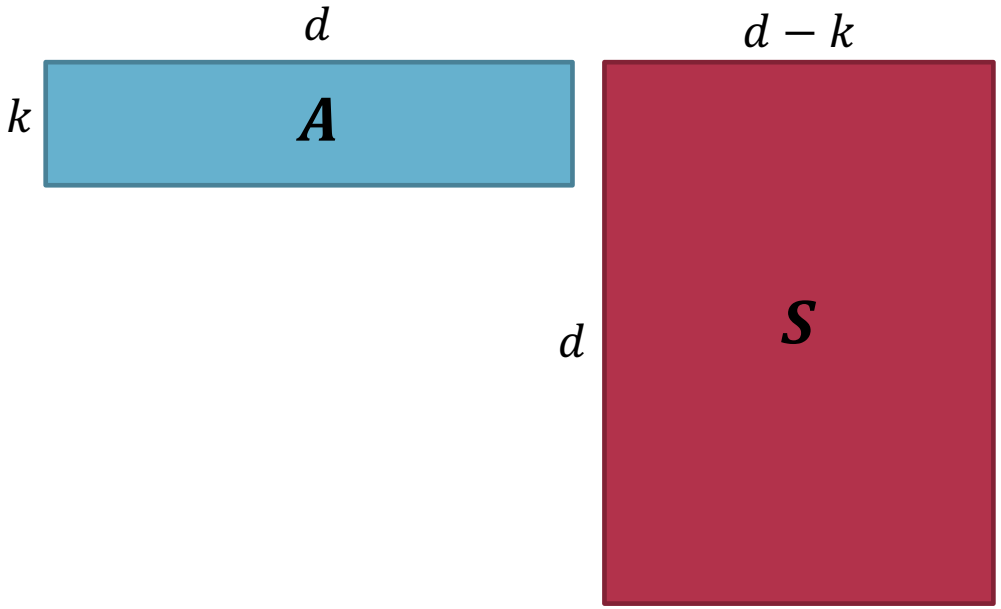

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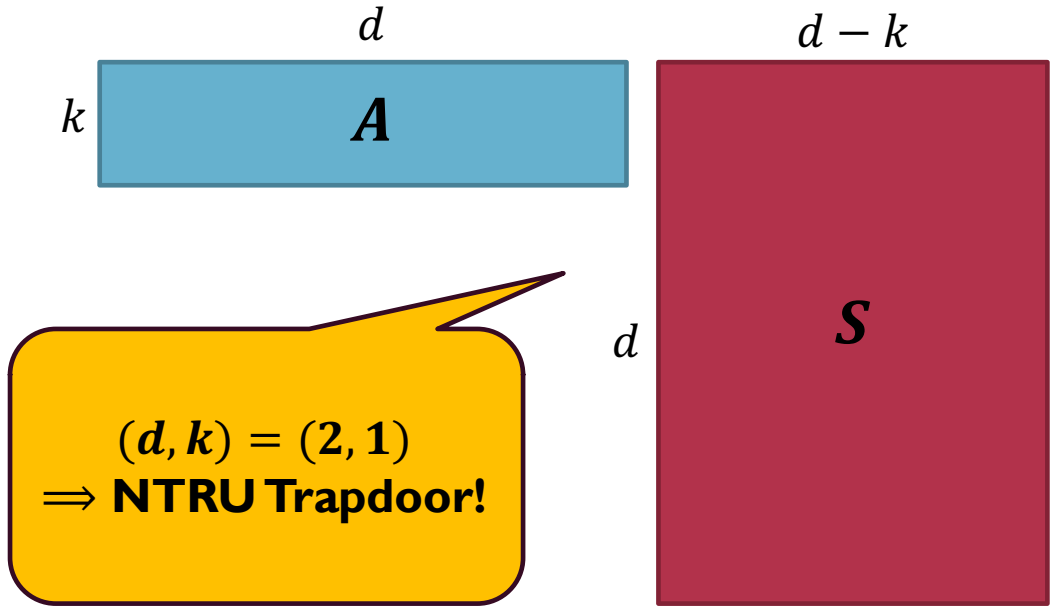

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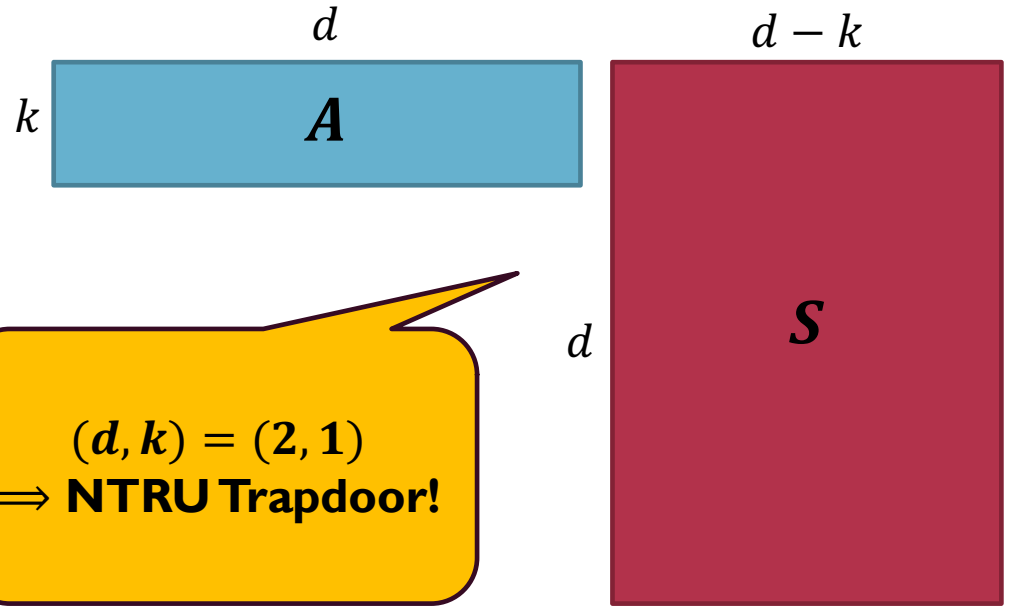

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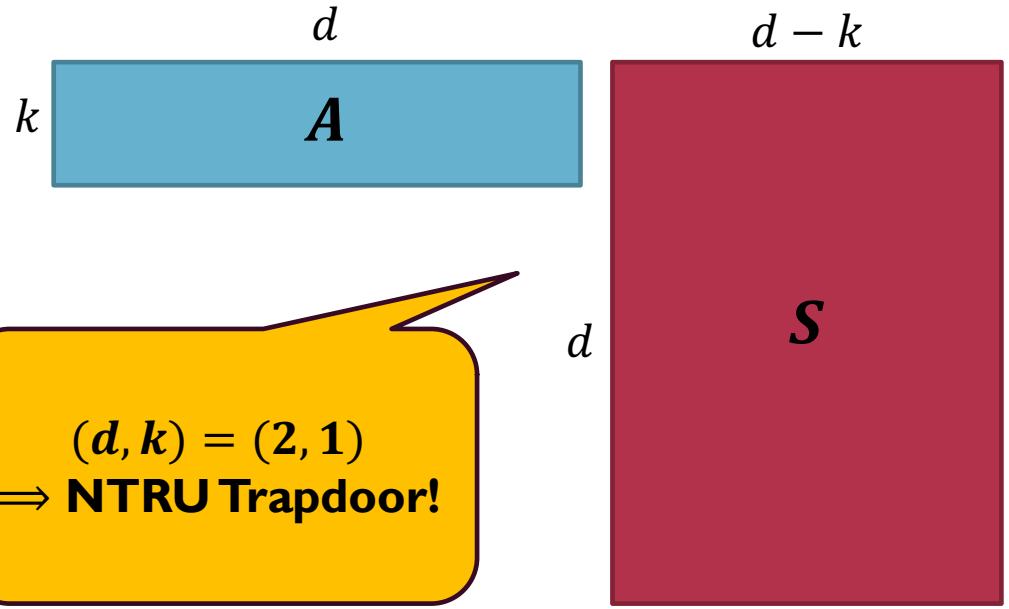

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$(\boldsymbol{d}, \boldsymbol{k}) = (\mathbf{2}, \mathbf{1})$
$\Longrightarrow$ **NTRU Trapdoor!**

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$\boldsymbol{fG - Fg = q}$

Our Idea

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3. Compute "short" (F, G, H) satisfying $\det \begin{pmatrix} f_1 & f_2 & F \\ g_1 & g_2 & G \\ h_1 & h_2 & H \end{pmatrix} = q$

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Multi-instance NTRU assumption

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How to compute "short" (F, G, H) **satisfying** $det\begin{pmatrix} f_1 & f_2 & F \\ g_1 & g_2 & G \\ h_1 & h_2 & H \end{pmatrix} = q$ **?**

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- Imitate the NTRU trapdoor computing "short" (F, G) s.t. $fG - Fg = det\begin{pmatrix} f & F \\ g & G \end{pmatrix} = q$

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How to compute "short" (F, G, H) satisfying $\boldsymbol{det}\begin{pmatrix} \boldsymbol{f_1} & \boldsymbol{f_2} & \boldsymbol{F} \\ \boldsymbol{g_1} & \boldsymbol{g_2} & \boldsymbol{G} \\ \boldsymbol{h_1} & \boldsymbol{h_2} & \boldsymbol{H} \end{pmatrix} = \boldsymbol{q}$?

- Imitate the NTRU trapdoor computing "short" (F, G) s.t. $fG - Fg = det\begin{pmatrix} f & F \\ g & G \end{pmatrix} = q$
  - Assume that $res(f) = \prod f(x^{2i+1}) \in Z$ and $res(g)$ are coprime

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How to compute "short" (F, G, H) satisfying $\boldsymbol{det}\begin{pmatrix} \boldsymbol{f_1} & \boldsymbol{f_2} & \boldsymbol{F} \\ \boldsymbol{g_1} & \boldsymbol{g_2} & \boldsymbol{G} \\ \boldsymbol{h_1} & \boldsymbol{h_2} & \boldsymbol{H} \end{pmatrix} = \boldsymbol{q}$?

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  - Assume that $res(f) = \prod f(x^{2i+1}) \in Z$ and $res(g)$ are coprime
  - $\alpha \cdot res(f) - \beta \cdot res(g) = 1 \Longrightarrow \left(q\alpha \cdot \prod_{i \neq 0} f\left(x^{2i+1}\right)\right) \cdot f(x) - \left(q\beta \cdot \prod_{i \neq 0} g\left(x^{2i+1}\right)\right) \cdot g(x) = q$

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  - $\alpha \cdot res(f) - \beta \cdot res(g) = 1 \Rightarrow \underbrace{\left(q\alpha \cdot \prod_{i \neq 0} f\left(x^{2i+1}\right)\right)}_{:= G_0} \cdot f(x) - \underbrace{\left(q\beta \cdot \prod_{i \neq 0} g\left(x^{2i+1}\right)\right)}_{:= F_0} \cdot g(x) = q$

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  - Reduce (F_0, G_0) with a "short" vector (f, g), and output (F, G)

Our Idea – Generalization

How to compute "short" (F, G, H) satisfying $det\begin{pmatrix} f_1 & f_2 & F \\ g_1 & g_2 & G \\ h_1 & h_2 & H \end{pmatrix} = q$?

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  - Assume $\gcd(res(f_1 g_2 - f_2 g_1), res(g_1 h_2 - g_2 h_1), res(h_1 f_2 - h_2 f_1)) = 1$

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How to compute "short" (F, G, H) satisfying $\boldsymbol{det}\begin{pmatrix} \boldsymbol{f_1} & \boldsymbol{f_2} & \boldsymbol{F} \\ \boldsymbol{g_1} & \boldsymbol{g_2} & \boldsymbol{G} \\ \boldsymbol{h_1} & \boldsymbol{h_2} & \boldsymbol{H} \end{pmatrix} = \boldsymbol{q}$?

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  - Assume $\gcd(res(f_1g_2 - f_2g_1), res(g_1h_2 - g_2h_1), res(h_1f_2 - h_2f_1)) = 1$
  - Obtain (F_0, G_0, H_0) satisfying $F_0 \cdot (g_1h_2 - g_2h_1) + G_0 \cdot (h_1f_2 - h_2f_1) + H_0 \cdot (f_1g_2 - f_2g_1) = q$

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- Our Case
  - Assume $\gcd(res(f_1g_2 - f_2g_1), res(g_1h_2 - g_2h_1), res(h_1f_2 - h_2f_1)) = 1$
  - Obtain (F_0, G_0, H_0) satisfying $F_0 \cdot (g_1h_2 - g_2h_1) + G_0 \cdot (h_1f_2 - h_2f_1) + H_0 \cdot (f_1g_2 - f_2g_1) = q$
  - Reduce (F_0, G_0, H_0) with "short" vectors (f_1, g_1, h_1) and (f_2, g_2, h_2)

Ongoing Works and Expectation

Ongoing Works and Expectation

- Analysis
  - Cryptanalysis on Multi-instance NTRU
  - Analysis on Signature size

- Expectation
  - Better Flexibility on Parameters than Falcon
  - Generalization of specific NTRU Trapdoor
  - What else..?

thank you!