Real-HEAAN: Approximate Homomorphic Encryption over the Conjugate-invariant Ring

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Contributions of Real-HEAAN

- An approximate Homomorphic Encryption of which the plaintext space is (purely) real number field

$\Longrightarrow$ NO waste of the plaintext space for real-number arithmetic contrary to HEAAN

$\Longrightarrow$ Prevent the potential problem of HEAAN

- Real-HEAAN supports twice more parallel computations compared to HEAAN under the same security level, speed, and memory (with new NTT method)

An Approxmiate HE Scheme
HEAAN

HEAAN: an Approximate HE scheme

HEAAN: **H**omomorphic **E**ncryption for **A**rithmetic over **A**pproximate **N**umbers

- Proposed by Cheon-Kim-Kim-Song in Asiacrypt'17
- Natural fit in real-world applications which require approximate computations of real numbers
- Abandoning exact computations, it gains a lot of advantages in efficiency:

  Ctxt/Ptxt expansion ratio, # Ptxt slots, rounding operation (for free)

HEAAN: an Approximate HE scheme

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The Decryption Circuit!
(No Additional Modulo Operation)

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Evaluation error + Decryption error

Encoding/Decoding of HEAAN

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- Let ϕ be an isomorphism (induced by canonical embedding) from $\mathrm{C}^{\frac{n}{2}}$ to $\mathrm{R}[X]/(X^n + 1)$

  (C and R denote complex/real number fields resp.)

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  (C and R denote complex/real number fields resp.)

- Given $\frac{n}{2}$ complex numbers $z_1, z_2, \dots, z_{\frac{n}{2}}$ and a scaling factor $\Delta > 0$,

$$\mathbf{Ecd}\left(\boldsymbol{z_1, \dots, z_{n/2}; \Delta}\right) = \left\lfloor \boldsymbol{\Delta \cdot \phi\left(z_1, \dots, z_{n/2}\right)} \right\rceil := m$$

- The decoding process is very simple, just evaluating a half of m-th primitive roots of unities

$$\mathbf{Dcd(m; \Delta)} = \left(\frac{\mathbf{1}}{\boldsymbol{\Delta}} \cdot \boldsymbol{m(\zeta_{4i+1})}\right)_{\boldsymbol{0 \le i < n/2}}$$

- The scaling factor controls the Encoding/Decoding error

Impact of HEAAN to real-world

iDASH Privacy & Security Workshop

- A Privacy & Security workshop holding competitions on secure genome analysis
- One of 3 tasks: secure genome analysis based on HE (e.g., Logistic Regression, GWAS,…)

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- One of 3 tasks: secure genome analysis based on HE (e.g., Logistic Regression, GWAS,…)
- HEAAN-based solutions won the 1st place both on 2017 and 2018
- All the submitted solutions of HE-based secure GWAS computation used HEAAN!

Some Limitations of HEAAN

Limitations of HEAAN

1. The Waste of the Plaintext Space

- The Plaintext space of HEAAN is

$$\mathrm{R}[X]/(X^n + 1) \simeq \mathrm{C}^{\frac{n}{2}}$$

where R and C denote the real / complex number field respectively.

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where R and C denote the real / complex number field respectively.

- In real-number applications, we only use the subring $\mathrm{R}^{\frac{n}{2}}$ of the plaintext space $\mathrm{C}^{\frac{n}{2}}$!

Limitations of HEAAN

2. The Complex Explosion Problem

- In real-number applications, we only care about the real part of a plaintext.
- However, the complex part of a plaintext is "internally growing up" in every operation!

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Let $a, c \approx 2^p$ and $b, d \approx 2^r$ for $r \ll p$.

$$\Longrightarrow \frac{b}{a}, \frac{d}{c} \approx 2^{r-p} \;\&\; \frac{ad+bc}{ac-bd} \approx 2^{r-p+1}$$

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- The complex part essentially explodes in large-depth circuit evaluations

Real-HEAAN

The core idea of Real-HEAAN

Use the subring of the cyclotomic ring!

- The plaintext space of original HEAAN

$$\mathbb{R}[X]/(X^n + 1) \simeq \mathbb{C}^{\frac{n}{2}}$$

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$$\cup$$

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$$\mathbb{R}[X]/(X^n+1) \simeq \mathbb{C}^{\frac{n}{2}}$$

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$$\cup \qquad\qquad \cup$$

$$\mathbb{R}[X+X^{-1}]/(X^n+1) \simeq \mathbb{R}^{\frac{n}{2}}$$

Here $X^{-1} := -X^{n-1}$ denotes the inverse of X modulo X^n+1

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- Let $R' \coloneqq Z[X + X^{-1}]/(X^n + 1)$

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- Let $R' := Z[X + X^{-1}]/(X^n + 1)$
- Every element of Real-HEAAN is built over R' instead of $R = \mathrm{Z}[X]/(X^n + 1)$
  - Secret Key: sk = $(-s, 1) \in R_q'^2$ where $R_q' = Z_q[X + X^{-1}]/(X^n + 1)$
  - Public Key: pk = $(a, b = a \cdot s + e) \in R_q'^2$
  - Ciphertext of $m \in R'$: ct = $(r \cdot a + e_1, r \cdot b + e_2 + m) \in R_q'^2$

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  (Note that τ is just a simple domain-restriction of $\phi \Longrightarrow \tau = \phi|_{\mathbb{R}^{n/2}}$)

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$$\mathbf{Ecd}\left(\boldsymbol{x_1, \dots, x_{n/2}; \Delta}\right) = \left\lfloor \boldsymbol{\Delta \cdot \tau\left(x_1, \dots, x_{n/2}\right)} \right\rceil \coloneqq m$$

- The decoding process is exactly same with HEAAN:

$$\mathbf{Dcd(m; \Delta)} = \left(\boldsymbol{\frac{1}{\Delta} \cdot m(\zeta_{4i+1})}\right)_{\boldsymbol{0 \le i < n/2}}$$

Real-HEAAN vs HEAAN

Real-HEAAN vs HEAAN

Our Claim

Real-HEAAN over $Z[X+X^{-1}]/(X^{2n}+1) \approx$ HEAAN over $Z[X]/(X^{n}+1)$

w.r.t. Security, Ring operation speed, and memory

#Ptxt Slots: n vs $n/2 \Longrightarrow$ twice more Parallel Computations!

Security of Real-HEAAN

[Security Reduction] Real-HEAAN is IND-CPA secure under the hardness assumption of RLWE over the number field $K := \mathrm{Q}[X + X^{-1}]/(X^{2n} + 1)$ (of which the extension degree $[K: Q] = n$)

[Cryptanalysis] RLWE over the number field K resists all known algebraic attacks on RLWE so that the best known attack is essentially the general attacks on LWE of dimension n

Efficiency of Real-HEAAN

1. Memory

- Every element of $R_q{}' = Z_q[X + X^{-1}]/(X^{2n} + 1)$ is express as $a(X) = a_0 + \sum_{i=1}^{n-1} a_i\,(X^i - X^{2n-i})$ for $a_i \in Z_q$

  $\Longrightarrow$ $n \cdot \log q$ bits are required to store each element

2. Speed

- Number Theoretical Transform (NTT): mapping between $Z_q[X]/(X^m - 1) \simeq Z_q^m$ with $O(m \log m)$ complexity
- Current best NTT method for $R_q = Z_q[X]/(X^n + 1)$ asymptotically requires $O(n \log n)$ complexity
- Our new NTT method for $R_q{}' = Z_q[X + X^{-1}]/(X^{2n} + 1)$ also requires $O(n \log n)$ complexity!

NTT method for R_q'

- Assume q is a prime

Trivial Approach:

$$
\begin{tikzcd}
R_q' = Z_q[X + X^{-1}]/(X^{2n}+1) \arrow[r, "\text{embedding}"] & Z_q[X]/(X^{4n}-1) \arrow[d, "\textbf{NTT of dim } 4n"] \\
\text{(Computations over } Z_q^{4n}\text{)} & Z_q^{4n} \arrow[d, "\textbf{Inverse NTT of dim } 4n"] \\
R_q' = Z_q[X + X^{-1}]/(X^{2n}+1) & Z_q[X]/(X^{4n}-1) \arrow[l, "\text{Mod } X^{2n}+1"']
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NTT method for R'_q

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\end{tikzcd}$$

Requires
NTT of dimension $4n$!

NTT method for R'_q

Our New Approach:

- Find a "simply computable" invertible linear transformation from R'_q to $Z_q[X]/(X^n - 1)$

$$R'_q \xrightarrow[]{\text{Simply Computable Linear map}} Z_q[X]/(X^n - 1)$$

$$a(X) = a_0 + \sum_{i=1}^{n-1} a_i\,(X^i - X^{2n-i}) \longrightarrow \tilde{a}(X) = \sum_{i=0}^{n-1} \tilde{a}_i X^i$$

where $\widetilde{a_0} = a_0$ and $\tilde{a}_i = a_i \cdot w^i + a_{n-i} \cdot w^{i-n}$ for $1 \le i \le n-1$ (w: $4n$-th prim. root of unity mod q)

- The inverse mapping is also simply computable with $O(n)$ complexity

NTT method for R'_q

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Requires
NTT of dimension n!

Conclusion

- Real-HEAAN provides twice more parallel computations compared to the original HEAAN while preserving the same level of security, ring operation speed, and memory.
- In other words, with the same number of parallel computations, Real-HEAAN is asymptotically twice faster than the original HEAAN.
- Moreover, Real-HEAAN prevents the complex explosion problem of HEAAN.
- The generalization of our new NTT method would be very interesting open topic!

Table 1. Comparison of our scheme and HEAAN

Approximate HE	$\texttt{OurScheme}(2n, q)$	$\mathrm{HEAAN}(n, q)$
Number of plaintext slots	n	$n/2$
NTT dimension	n	n
Bit size of ciphertexts	$2n \log q$	$2n \log q$

thank you!

Homomorphic Encryption

Homomorphic Encryption

Homomorphic Encryption (HE) :

An Encryption scheme which allows computations on encrypted data

$$
\begin{tikzcd}
\mathrm{enc}(m) \arrow[r, "\mathrm{eval}"] & \mathrm{enc}(f(m)) \\
m \arrow[u, "\mathrm{enc}"] \arrow[r, "f"] & f(m) \arrow[u, "\mathrm{enc}"']
\end{tikzcd}
$$

Homomorphic Encryption

Homomorphic Encryption (HE) :

An arbitrary circuit over encrypted data can be evaluated w/o decryption!

Selected as 10 Emerging Technologies (MIT Technical Review 2011)

Ciphering: Gentry's system allows encrypted data to be analyzed in the cloud. In this example, we wish to add 1 and 2. The data is encrypted so that 1 becomes 33 and 2 becomes 54. The encrypted data is sent to the cloud and processed: the result (87) can be downloaded from the cloud and decrypted to provide the final answer (3). Credit: Steve Moors

Pros / Cons of HE

- **Pros**

- HE allows us to evaluate an arbitrary circuit (w/ bootstrapping)
- Data Leakage Prevention against hackers (w/o decryption key)
- Various Real-World Applications: Statistical Analysis, Searching, Machine Learning (over encrypted data)

- **Cons**

- Large Ciphertext/Plaintext Expansion ratio (40 ~ 1000 for FHE)
- Evaluation Speed: more than hundreds of times slower than one on unencrypted state

$\Longrightarrow$ **Individualized Optimization** is going on for each operation!

Various Lattice-based HE schemes

Scheme	Plaintext	Good	Bad	Library
Wordwise Encryption - Brakerski-Gentry-Vaikuntanathan'12 - Gentry-Halevi-Smart'12a,b,c - Brakerski'12, Fan-Vercauteren'12 - Halevi-Shoup'13,14,15	$GF(p^d)$ (Z_p)	Polylog overhead (Amortized time & Expansion rate)	Bootstrapping	HElib SEAL …
Linear Error growth & Quad. Ctxt size - Gentry-Sahai-Waters'13	Z, Z[X] ({0,1})	Toolkit for FHEW	Inefficient	-
Bitwise Encryption - Ducas-Micciancio'15 - Chillotti-Gama-Georgieva-Izabachene'16,17	{0,1}, $(\{0,1\}^*)$	Evaluation with Bootstrapping Latency	Amortized time & Expansion rate	FHEW TFHE

Application Researches on HE (2017 ~ Mar. 2018)

"Homomorphic Encryption" in ePrint and IEEE Xplore

Machine Learning:	11	(2018/233,202,139,074, 2017/979,715. SSCI, IEEE Access, IEEE Journal, ICCV, SMARTCOMP)
Neural Network:	2	(2018/073, 2017/1114)
Genomic Data:	7	(2017/955,770,294,228. EUSIPCO, SMARTCOMP, IEEE Journal)
Health Data:	2	(IBM Journal, IEEE Journal)
Biometric Data:	2	(IEEE Access, IEEE Conference)
Energy Management:	3	(2017/1212. IEEE Big Data, IET Journal)
Big Data:	1	(ICBDA)
Advertising:	1	(WIFS)
Internet of Things:	1	(IWCMC)
Election:	1	(2017/166)

The Construction of HEAAN

Idea 1: Every number contains an Approximation Error (from the unknown true value).

$\Longrightarrow$ Consider the error e of a ciphertext c as a part of the approximation error

$$c = \mathrm{Enc}(m) \quad \text{if} \quad \langle c, \mathrm{sk} \rangle \pmod{q} = m + e \approx m$$
$$(= m^*)$$

Simple Example:

$1.234 \Rightarrow$ (scale-up by $p = 10^4$) $\Rightarrow$ $12{,}340.$

$\Rightarrow$ (Encrypt) $\Rightarrow$ $[\langle c, \mathrm{sk} \rangle]_q$ = $12{,}344$ $\approx 1.234 \times 10^4$ $\Rightarrow$ (scale-down by p) $\Rightarrow$ 1.234

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The Decryption Circuit!
(No Additional Modulo Operation)

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The Construction of HEAAN

Idea 2: Approximate **Rounding** (ReScaling; RS) for (almost) Free!

- Assume that the secret key sk has sufficiently small coefficients.
- For a ciphertext c of the message m, define $c' = \lfloor p^{-1} \cdot c \rceil$.
- Then, it holds that

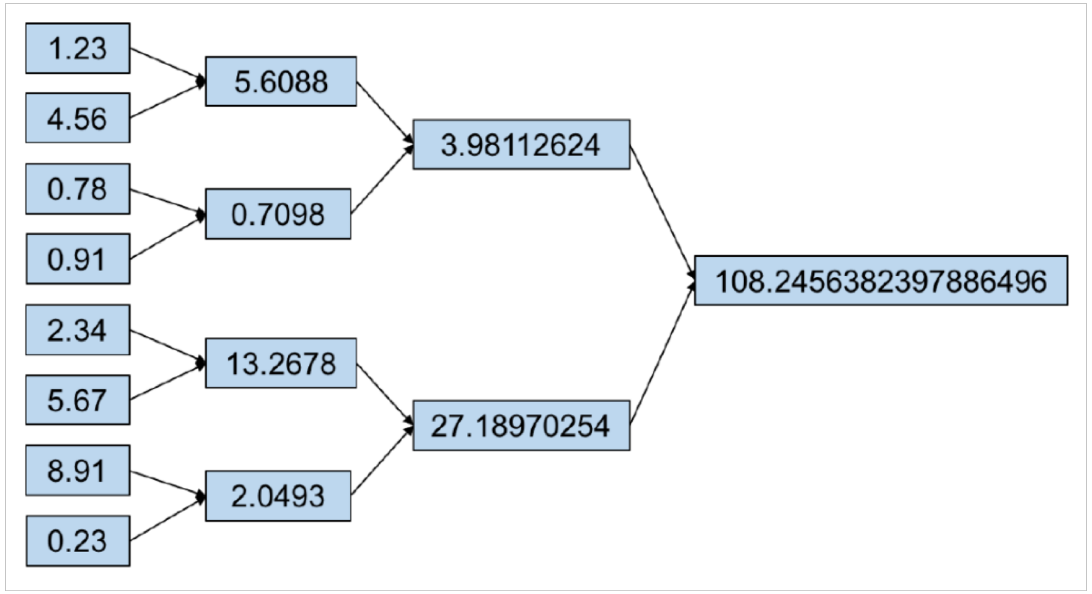
$$\langle c, \mathrm{sk} \rangle \pmod q = m^*$$

$$\Longrightarrow \langle c', \mathrm{sk} \rangle \pmod{\mathrm{p}^{-1} q} \approx p^{-1} m^* \text{ (an approximate rounding of } m^*\text{)}$$

- Rounding of a ciphertext directly derives an approximate rounding of the message!

Real Number Computations in HEs

Q1) What is the main problem of previous wordwise HEs in computation of real numbers?



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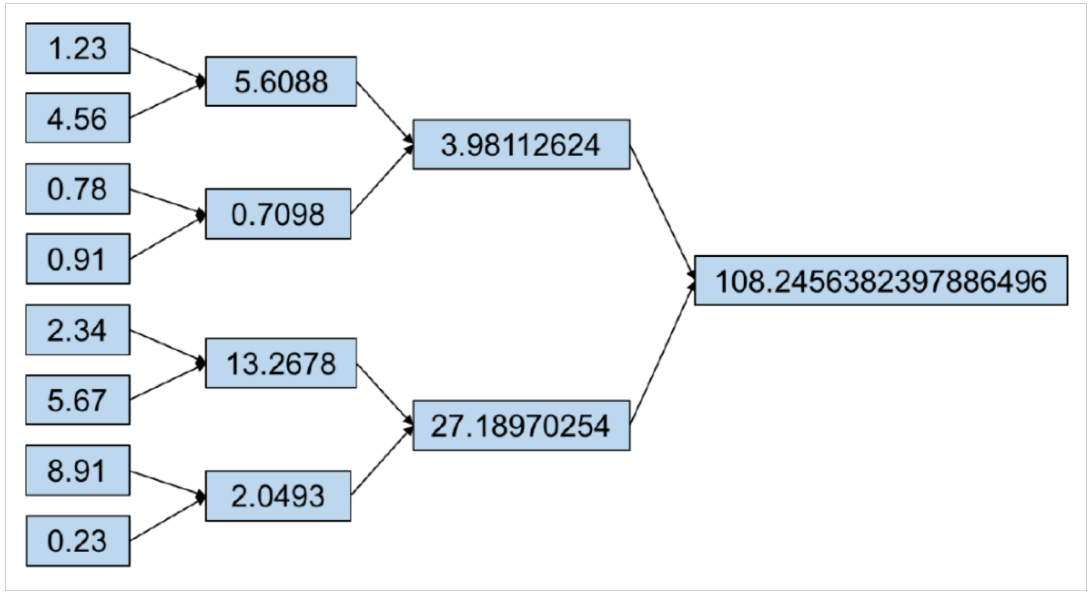
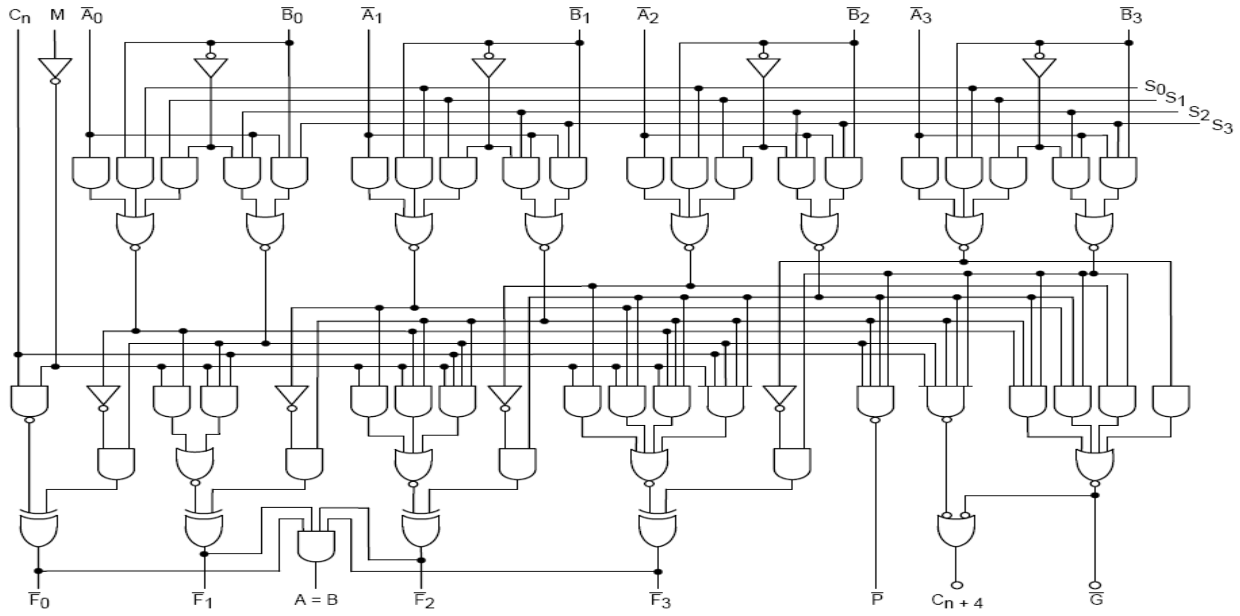


Ans) The exponential growth of the plaintext size (millions of bits after 20-depth multiplications)

- Ctxt size $\approx O(2^L)$, or other new techniques are required (L : level parameter)
- One solution is to extract MSBs and store them, but very expensive!

Real Number Computations in HEs

Q2) How about bitwise HE schemes?



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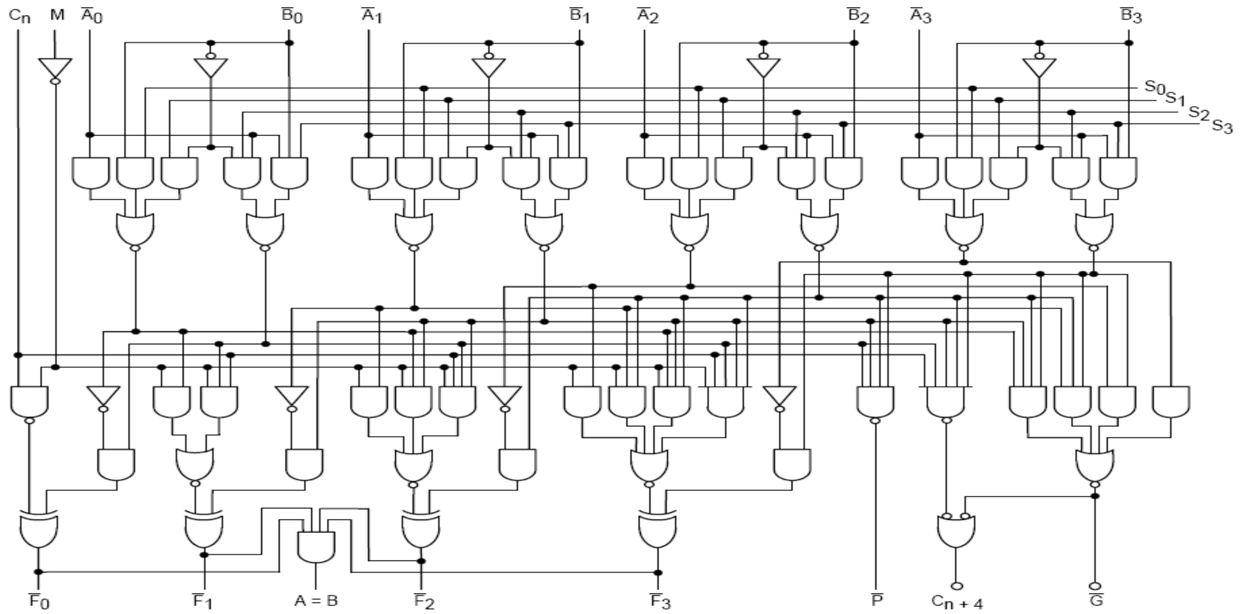
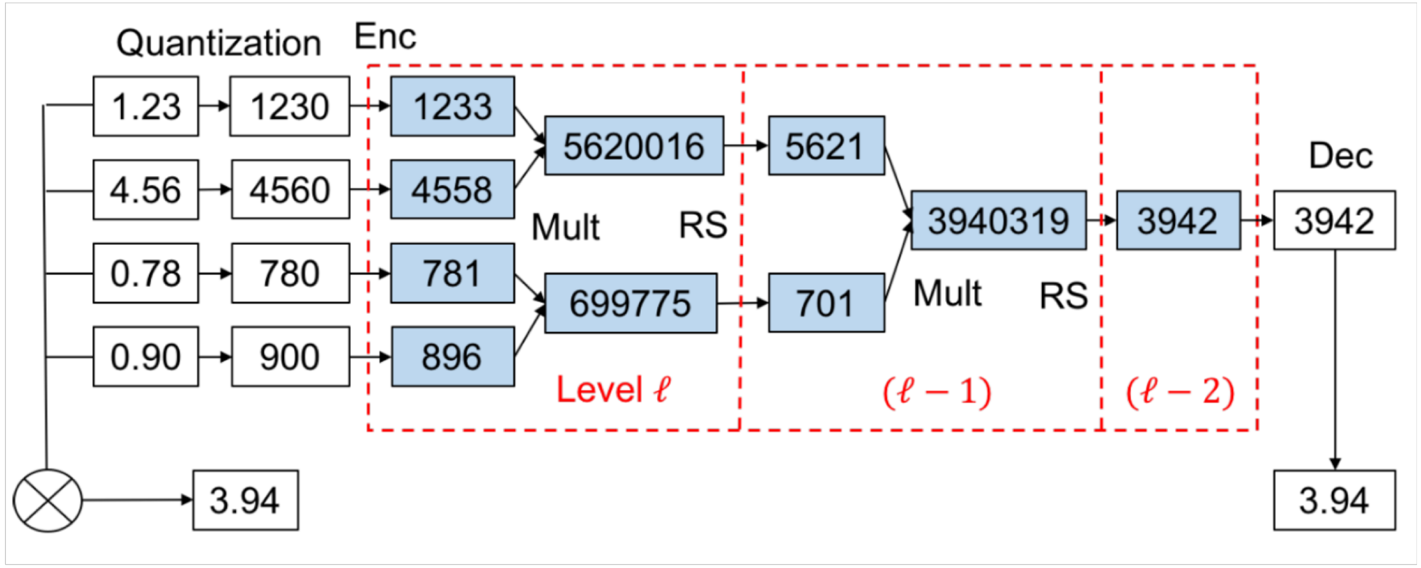


Ans) Too many gates required to represent an operation between large-precision numbers!

- 0.06 sec for (2-to-1) gate, 10 sec for (6-to-6) circuit.
- 75 gates for an operation between 4-bit strings.
- Then, how many gates for 16-bit / 32-bit precision multiplication?

Real Number Computations in HEAAN

Q3) Then, how does it work in HEAAN?



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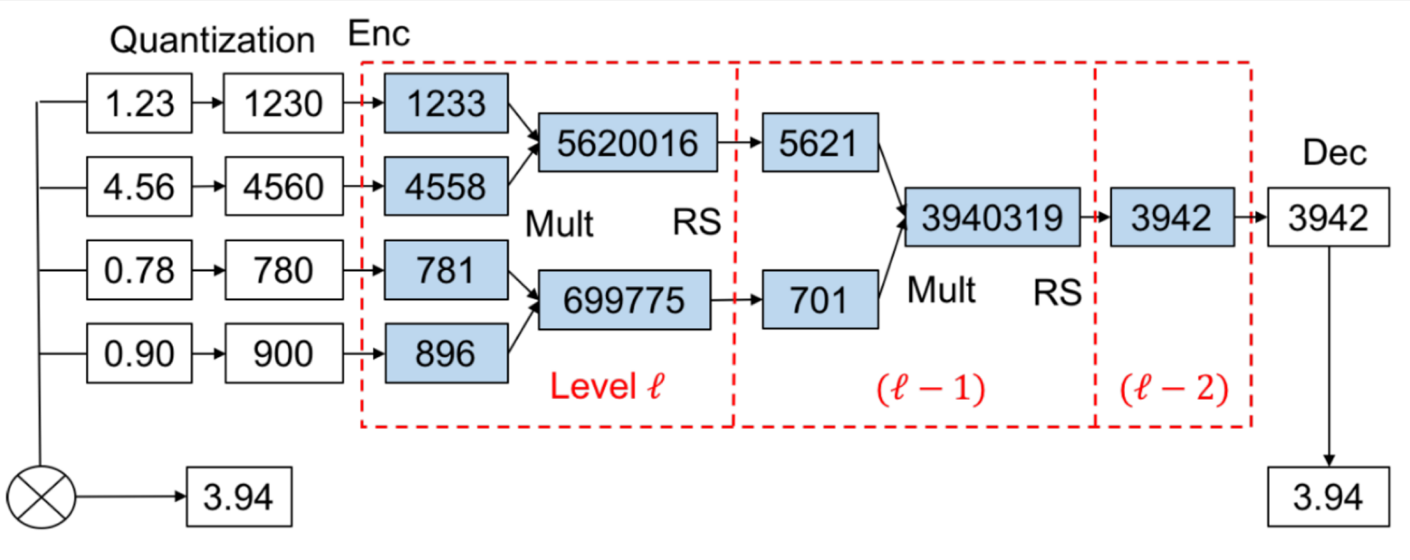

- Imitating the procedure of approximate arithmetic on computer system
- No additional cost for the "rounding"(RS) process!
- Ctxt size $\approx O(L)$ (L : level parameter), since HEAAN only stores most significant bits